Calculating Perfectly Paced Race Pace

Running a completely even split race is rare. Often an athlete's best effort may come at a championship race which may be tactical and have less than ideal splits for a fast time, or, as many beginner (and experienced) runners do, they may start the race too quickly accidentally. This is a problem as it is important for athletes and coaches to know how fast they can run for the sake of developing an appropriate training plan or goal setting. Here, I present a formula to calculate how fast the athlete could have run if they had run perfect splits. The formula works by taking in the race splits, calculating how much 'fatigue' an athlete would get from running that split, summing up the total 'fatigue' and then figuring out what evenly paced race pace would give the same amount fatigue.

Assumptions

The formula uses two assumptions. The first assumption is that running a set percentage of a race gives an equivalent fatigue regardless of the race distance. This means running 5km at 10km race pace is equivalent in terms of fatigue to running 2.5km at 5km race pace or 1.5km at 3km race pace.

The second assumption is a pace conversion formula. We will use a variation of "Frank Horwill's 4 second rule". This *rule* states that if you go up in an event distance the athlete should be able to complete the race at run 4 seconds slower per lap than the original race distance pace. This means a 3:45 1500m runner (60s/lap) should be able to run 8:00 (64s/lap) for the 3000m. This rule can be improved by using the athlete's approximate mile time (divided by 60) as how much the pace should be changed per lap. This is done because slower athletes tend to have a larger change in pace for a longer race event. The next change we can make is instead of saying "every time you increase in race distance you slow down by f second" you can say "every time you double the race distance you slow down by f seconds". This allows us to create the formula below that will accommodate any initial distance, ending distance or change in pace.

$$\Delta P = f * \log_2 \frac{D_n}{D_i}$$

$$D_n = D_i 2^{\left[\frac{\Delta P}{f}\right]}$$

Where D_n is the new distance, D_i is the initial distance, ΔP is the change in race pace and f is the athletes approximate mile time

While both assumptions have their flaws. The first assumption, running 400m at 800m pace does not feel equally as fatiguing as running a half marathon at marathon pace. Similarly for the second assumption, there are very few runners who can run an equivalent time over a wide range of distances (800m to marathon). However, both assumptions should be reasonable if the race pace (or distances) are close.

Calculations

The formula works by first calculating how far the athlete could have run if they ran at their overall race pace for the whole race, instead of uneven splits. This is done using the formula based on "Frank Horwill's 4 second rule" for each race split we are given.

$$D_{e,i} = D_i 2^{\left[\frac{\Delta P_i}{f}\right]}$$

Where $D_{e,i}$ is the effective distance, D_i is the actual distance they ran, $\Delta P_i = P_t - P_i$ where P_t is the pace that the athlete ran for the entire race and P_i is the pace the athlete ran for that portion of the race.

By summing the effective distance of all portions of the athlete's race ($D_n = \sum_i D_{e,i}$) we get a new, longer, distance that the athlete could have run at pace P_t. This new distance can then be converted, using a rearranged form of the above formula back to the original distance to get how fast the athlete could run for a perfect split race.

$$\Delta P = f \log_2 \left[\frac{D_n}{D_T}\right]$$

Which, using the original variables gives us the following formula:

$$\Delta P = f \log_2 \left[\frac{\sum_i \left(D_i 2^{\left[\frac{\Delta P_i}{f}\right]}\right)}{\sum_i (D_i)}\right]$$

Example

Lets say an athlete ran a 9min 3km. Their kilometre splits were: 2:50, 3:00, 3:10. Lets set $f = 10 \frac{s}{km}$ to make the calculations easier, for better results f should be closer to $f = 11.25 \frac{s}{km}$ for a 9min 3km runner. Since the first kilometre is run at the athletes 1500m pace (10sec faster per kilometre), they get 2km of effective fatigue.

$$D_{e,0} = D_0 2^{\left[\frac{\Delta P_0}{f}\right]}$$

$$D_{e,0} = 1km\ 2^{\left[\frac{10}{10}\right]} = 2km$$

The second kilometre is run at the same pace as the entire race, so the effective distance is equal to the actual distance $D_{e,1} = 1km$.

The final kilometre is run 10 seconds slower than the race pace (the athletes 6km pace). This gives the following effective fatigue:

$$D_{e,2} = D_2 2^{\left[\frac{\Delta P_2}{f}\right]} 1km\ 2^{\left[\frac{-10}{10}\right]} = 1km\ 2^{-1} = 0.5km$$

Summing all the effective distances gives $D_n = 2km + 1km + 0.5km = 3.5km$. This means that the athlete, had they run even splits, could have run 3.5km at their total pace (3:00min/km). To adjust this to a 3km time we can use the formula:

$$\Delta P = f \log_2 \left[\frac{D_n}{D_T}\right]$$

$$\Delta P = 10 \log_2 \left[\frac{3.5km}{3km}\right] = 2.2sec$$

Therefore, the athlete could have run 2.2sec/km faster or run 8:53 instead of 9:00.